A Novel Multi-Temporal DInSAR Phase Unwrapping Algorithm Based On Compressive Sensing and Minimum Cost Flow Techniques

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FRINGE 2023
University of Leeds, UK | 11-15 September 2023

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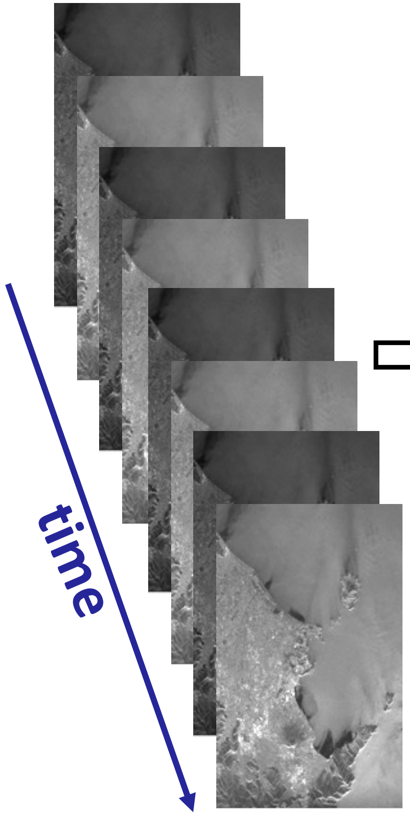
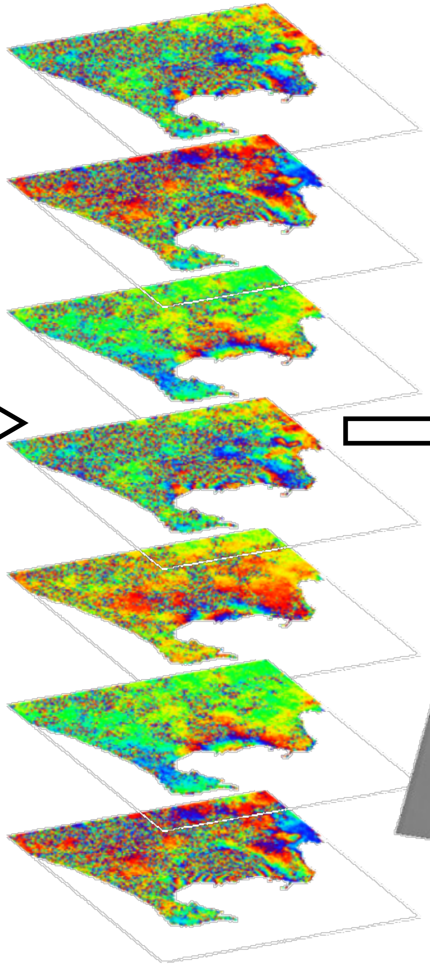
Small BAseline Subset (SBAS) approach

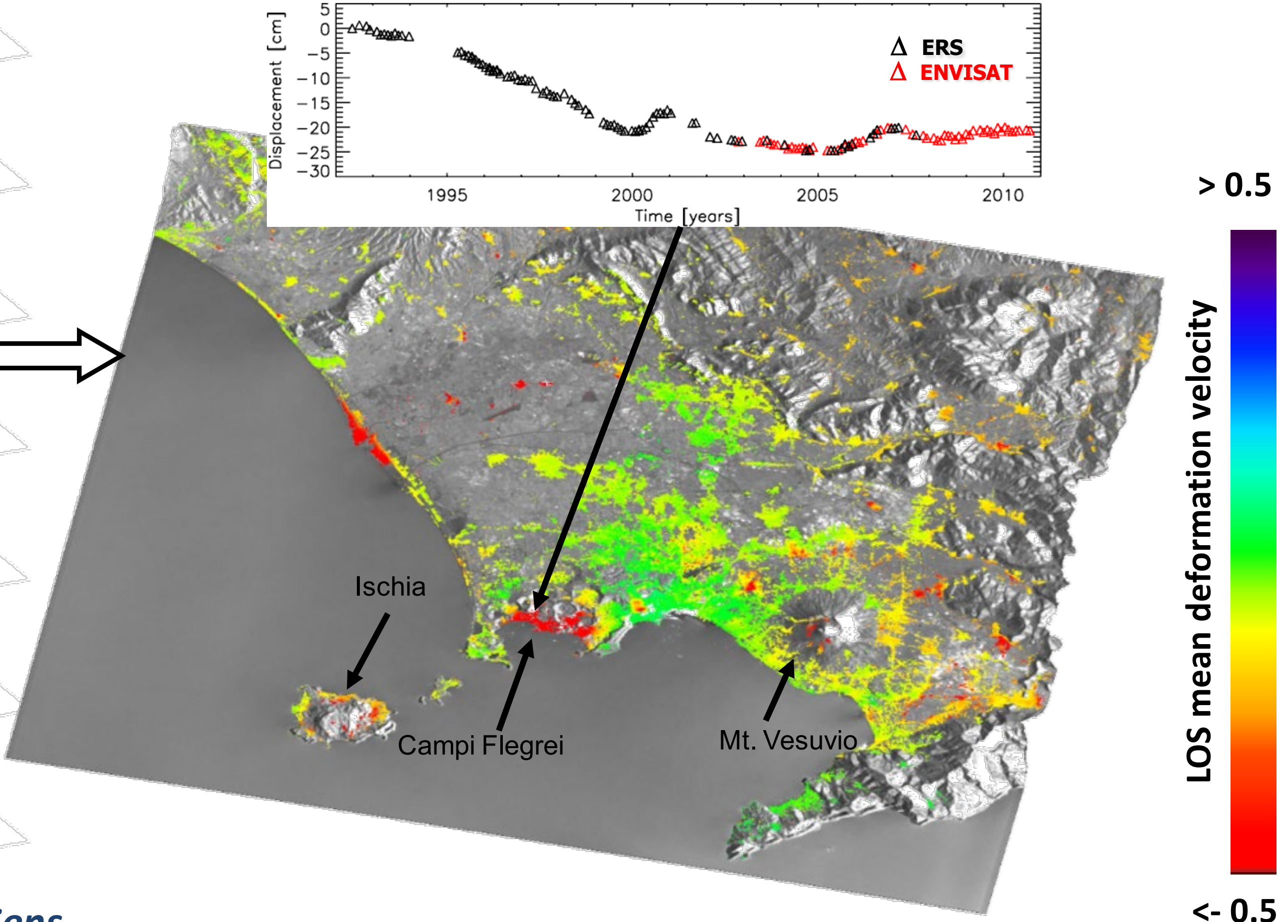
SAR Images



SB Interferograms



ERS/ENVISAT images (1992 – 2010)



Berardino et al., 2002, IEEE Trans. Geosci. Remote Sens.
Pepe et al., 2005, IEEE Trans. Geosci. Remote Sens.

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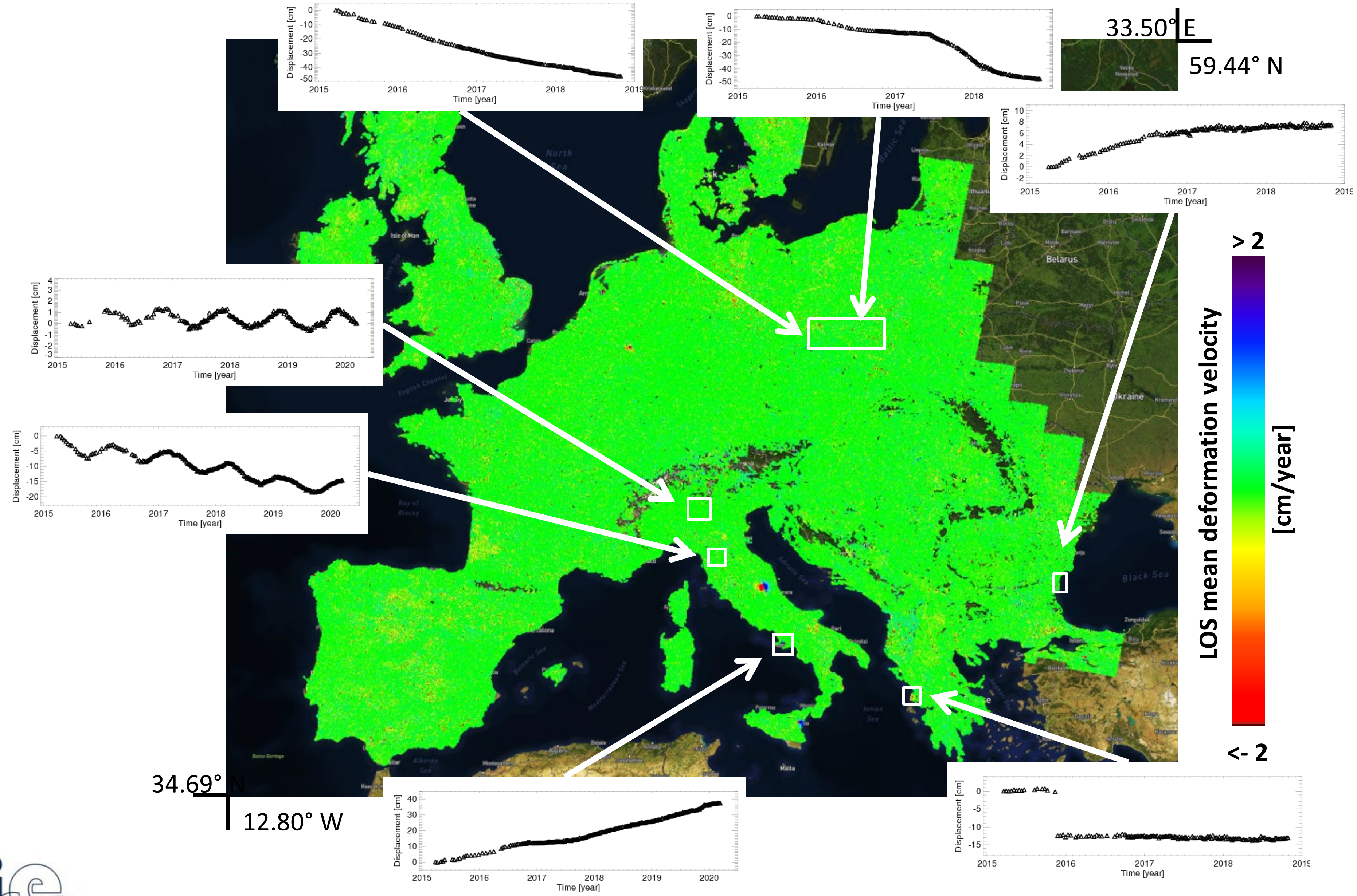
Parallel SBAS medium resolution analysis at continental scale



EUROPEAN PLATE OBSERVING SYSTEM

Casu et al., 2014, IEEE JSTARS
Manunta et al., 2019, IEEE TGRS
Lanari et al., 2020, Rem. Sensing

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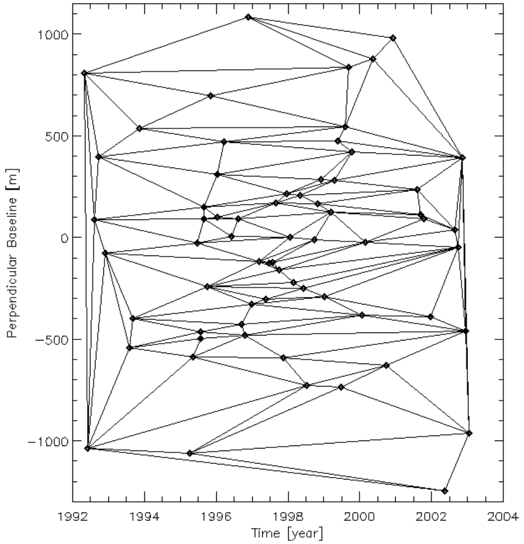
Extended Minimum Cost Flow (EMCF) PhU algorithm

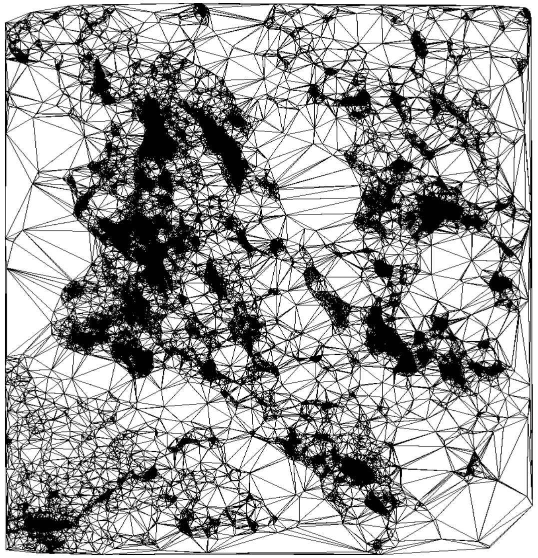
The EMCF PhU algorithm is carried out via a two-step processing procedure:

- Temporal PhU step (MCF PhU)
- Spatial PhU step (MCF PhU)

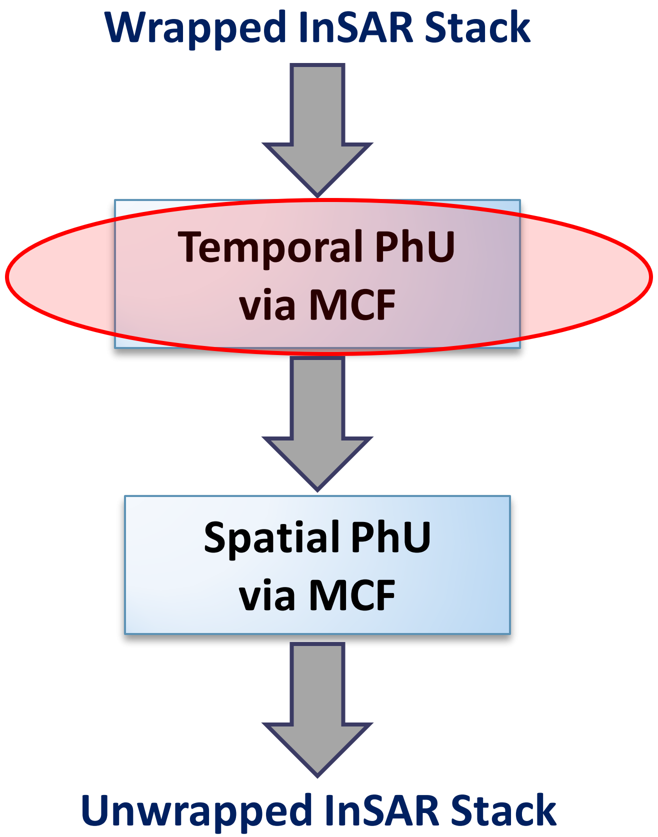
In order to perform temporal and spatial PhU procedures, **two Delaunay triangulation** networks are considered:



Temporal/Spatial baseline plane



Azimuth/Range plane



Pixels to be unwrapped are evaluated in the Az/Rg plane through the triangular coherence:

$$\Upsilon(P) = \frac{1}{\Lambda}\left|\sum_{k=1}^{\Lambda} exp\left[j\Delta\phi^{tr}_{res_k}(P)\right]\right| > thres$$

Pepe and Lanari., 2006, IEEE TGRS

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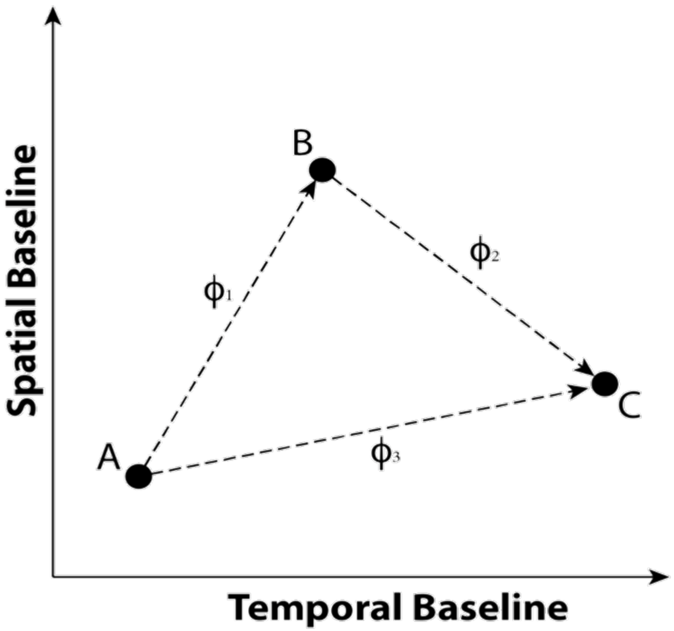
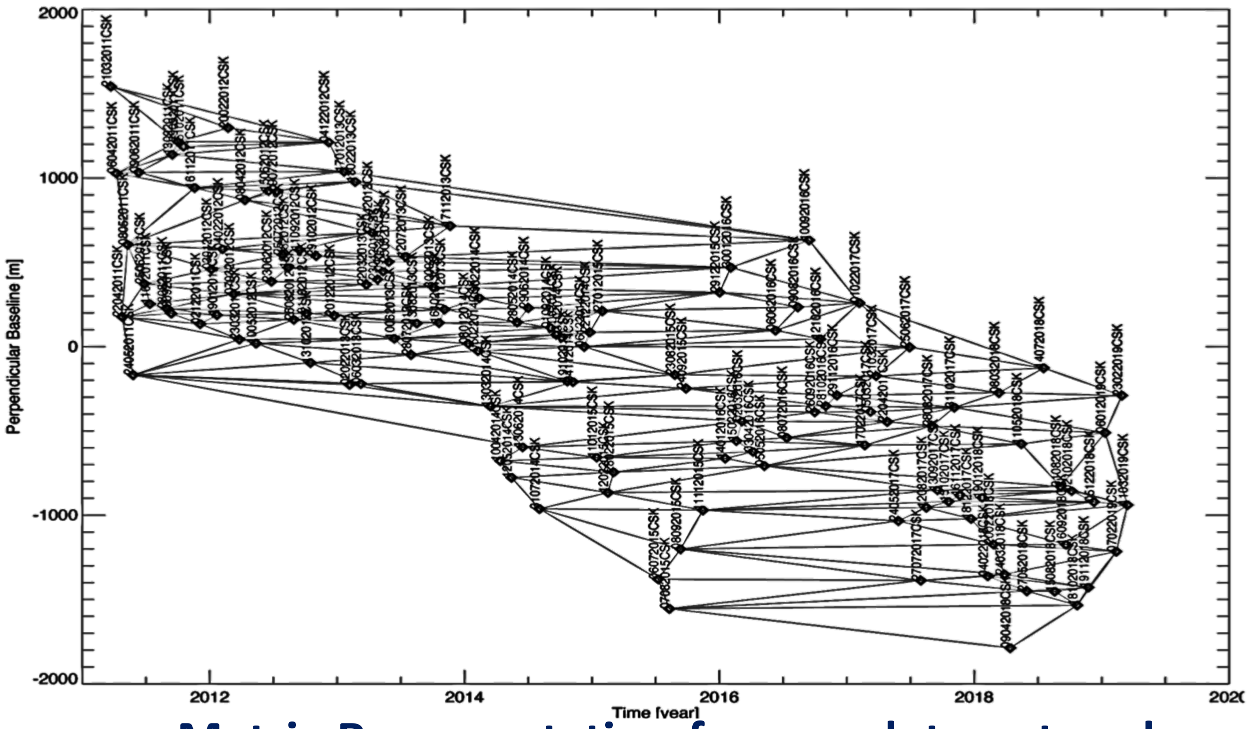
Phase closure

Interferometric triangle



DInSAR Network



Phase Closure Property

- In case of **correctly unwrapped phases**, for any point located at an arbitrary position (x,r), we can write:

$$\boldsymbol{\phi_1(x,r) + \phi_2(x,r) - \phi_3(x,r) = 0}$$

Matrix Representation for complete network

- For all triangles in Interferometric network, we have:

$$\boldsymbol{C\phi = 0}$$

where, C is of order $\boldsymbol{Q} \times \boldsymbol{M}$ and $\boldsymbol{Q} < \boldsymbol{M}$

- ***Q*** and ***M*** are the total number of triangles and interferograms of the network, repectively.

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Compressive Sensing and Phase Unwrapping

What happens if phases are not unwrapped?

- Violation of irrotational property over a subset of interferograms:

$$C\widehat{\phi} \neq 0$$

Equivalently:

$$C\left(\widehat{\phi} - 2\pi K\right) = 0$$

$$\boxed{CK = \frac{1}{2\pi} C\widehat{\phi} = \phi^{tr}}$$

- Where $\epsilon \in \mathbb{Z}^{1\times M}$ is the vector of PhU corrections.
- A physically sound solution is the one with a minimum number of 2π multiples.

Compressive Sensing (CS)

- Reconstructs signals from set of under sampled noisy measurements.
- Effectively solves inverse problems characterized by sparse solutions.
- Solves $L_0 -$ norm problems through $L_1 -$ norm minimization approaches.

CS theory

L_0 Optimization

$$Arg\,min\|K\|_0 \text{ s.t. } CK = \phi^{tr}$$

- No Mathematical Representation
- Computationally inefficient

L_1 Optimization

$$Arg\,min\,\|K\|_1 \text{ s.t. } CK = \phi^{tr}$$

- **Several algorithms to solve it (i.e., IRLS)**
- **Computationally efficient**

M. Manunta and Y. Muhammad., 2021, IEEE Trans. Geosci. Remote Sens.

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CS-Based EMCF approach

Step-1: Temporal Phase Unwrapping

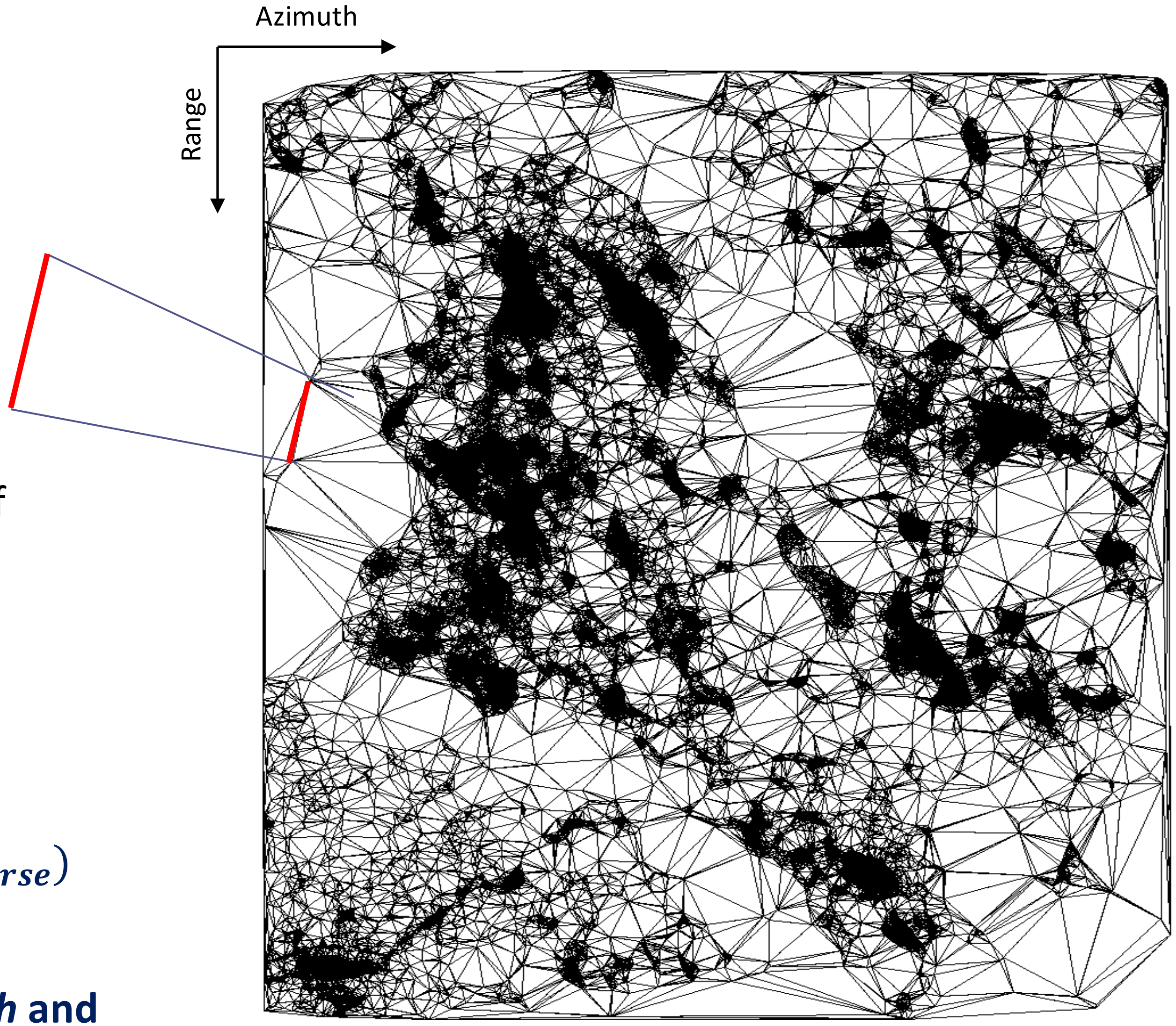
- **Goal**: Estimating the unwrapped phase relevant to each spatial arc (in azimuth/range plane)
- This operation is performed using L1 optimization approach (IRLS method is applied)

Step-2 : Spatial Phase Unwrapping

- **Goal**: Performing spatial phase unwrapping operation of each interferogram using MCF approach.
- Each arc is initialized to the value provided by temporal PhU.
- The cost of each arc is based on the result of the temporal algorithm.

✓ $$Cost = \begin{cases} 1 & (\gamma_{rnd} < th) \mid (N_{corr} > th_{sparse}) \\ 100,000 & \text{otherwise} \end{cases}$$

Spatial PhU can be iterated several times by changing *th* and *th*$_{sparse}$ to retrieve more solutions to be averaged in one final result



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Spatial PhU CS-Based EMCF approach

- ✓ $Cost = \begin{cases} 1 & (\gamma_{rnd} < th) \mid (N_{corr} > th_{sparse}) \\ 100{,}000 & \text{otherwise} \end{cases}$

- ➢ The higher the cost, the higher is the confidence on the estimated solution.

- ✓ $N_{corr} = \text{Number of nonzero elements} = ||K_{rnd}||_0$

Cost_1 K_{rnd} → Spatial PhU ϕ_1 → SVD → Phase Residuals φ_1^{res} → Weights

Cost_2 K_{rnd} → Spatial PhU ϕ_2 → SVD → Phase Residuals φ_2^{res} → Weights

....

Cost_N K_{rnd} → Spatial PhU ϕ_N → SVD → Phase Residuals φ_N^{res} → Weights

$$w_i = 1 - \frac{|\varphi_i^{res}|}{\pi}$$

$$\phi = \frac{\sum \phi_i w_i}{\sum w_i}$$

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Sentinel-1 Etna test site

- ✓ Largest active volcano in Europe.
- ✓ Data acquired by Sentinel-1 (descending orbit, Track 124) between 2015 and 2019.
- ✓ 227 S-1 Acquisitions
- ✓ 638 Interferograms

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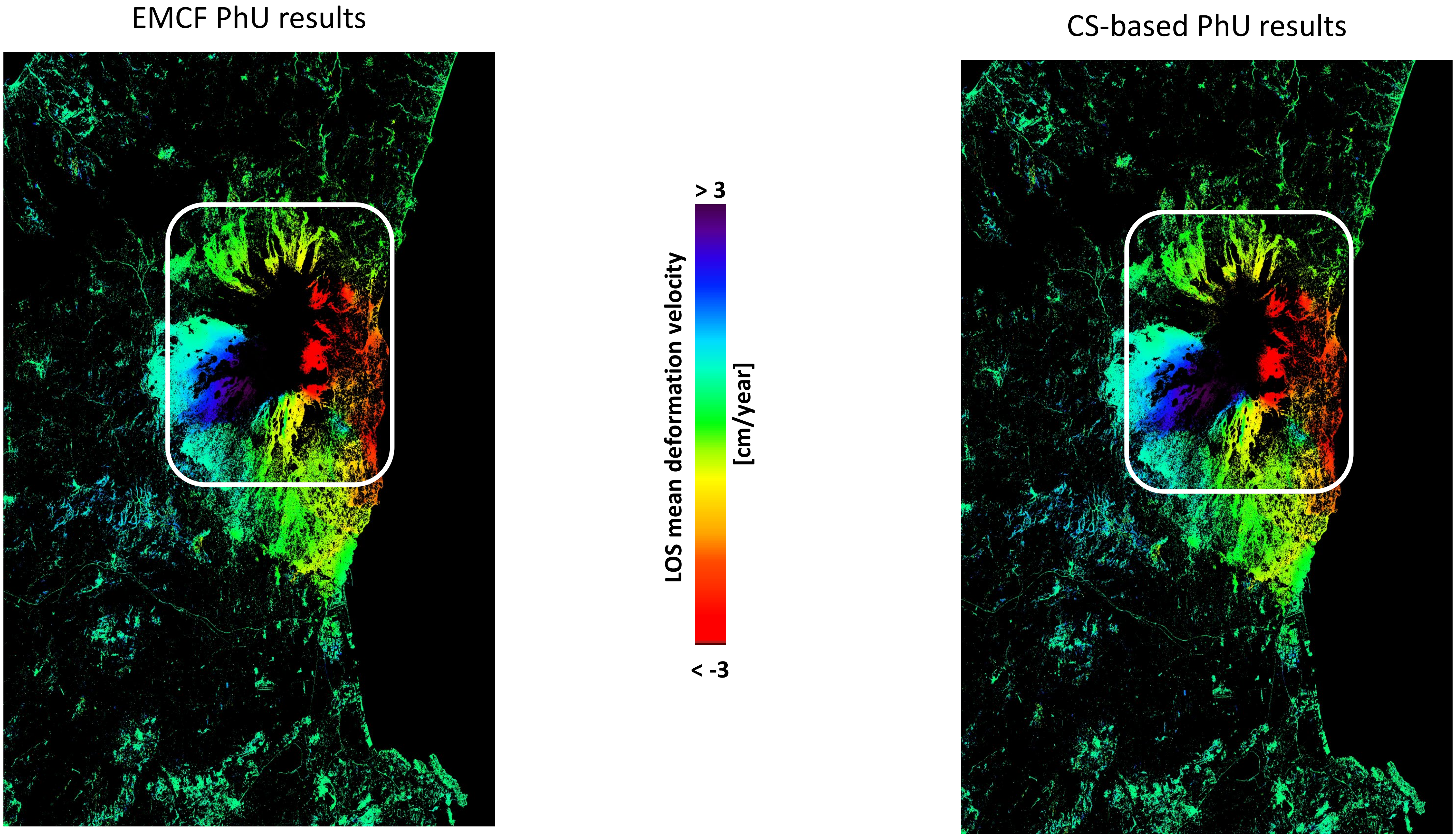
Sentinel-1 Etna test site



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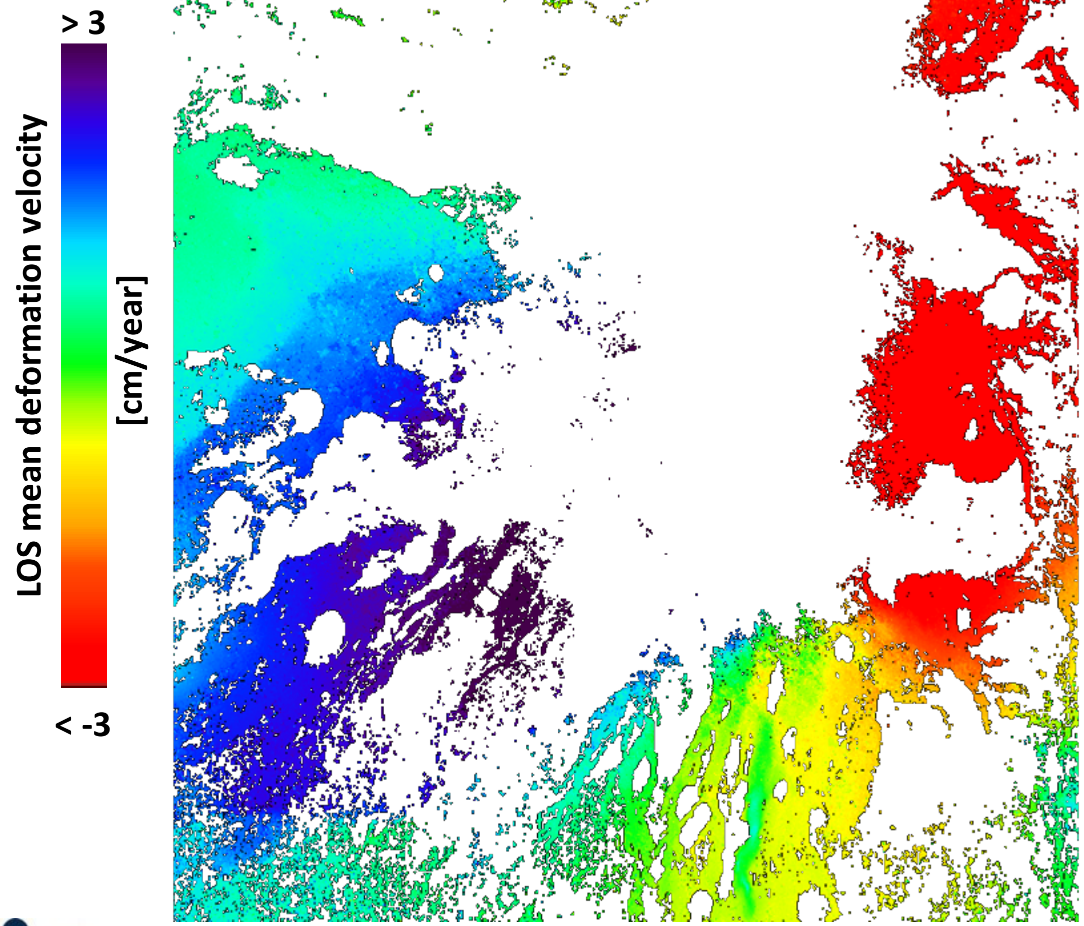
EMCF PhU Results on Sentinel-1 Etna test site



EMCF PhU Results

- 882,132 investigated points
- 818,816 coherent points (92.8%) (coherence > 0.9)

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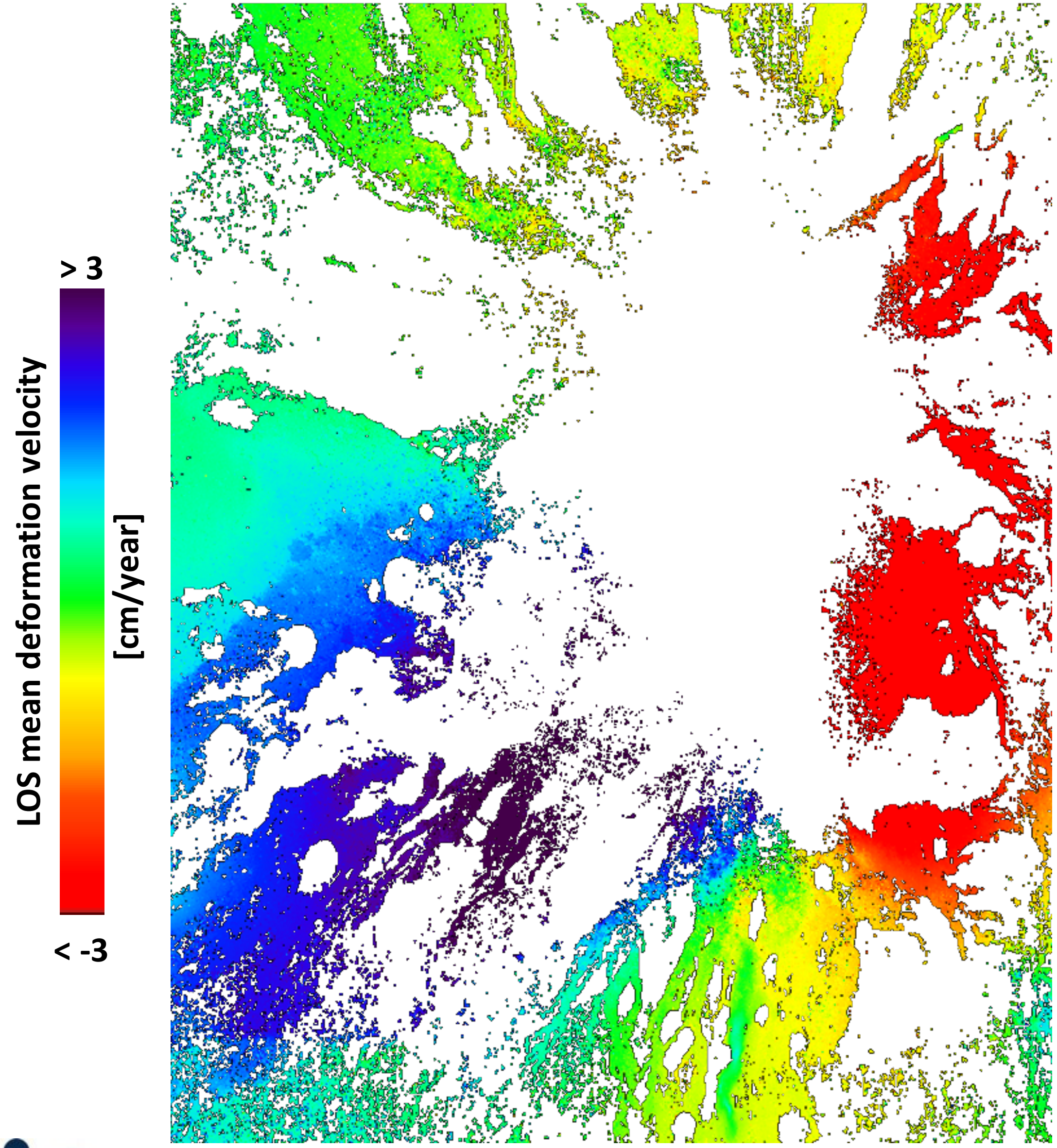
CS-based PhU Results on Sentinel-1 Etna test site



CS-based PhU Results

- 882,132 investigated points
- 838,944 coherent points (95.1%) (coherence > 0.9)

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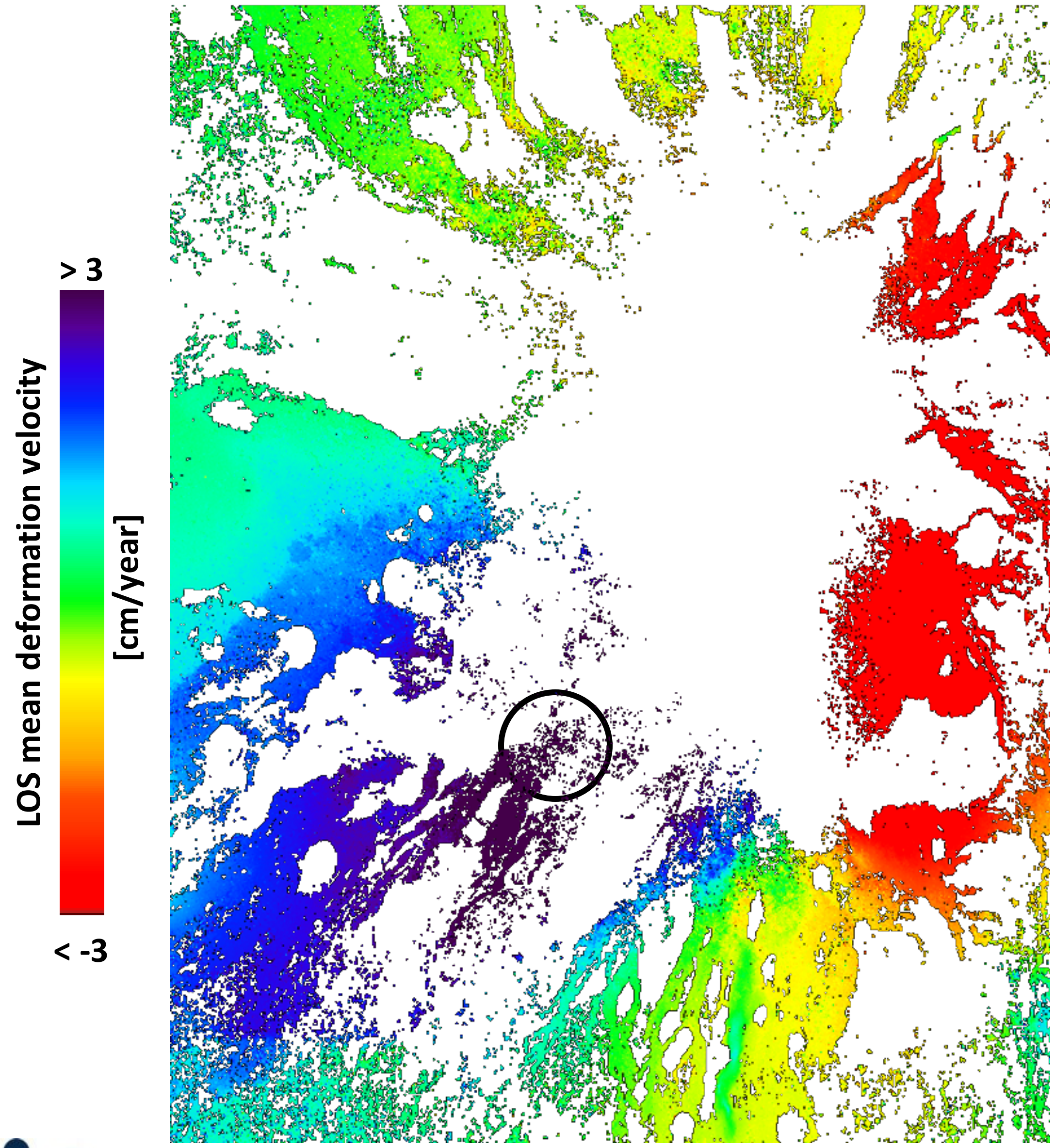
CS-based PhU Results on Sentinel-1 Etna test site



CS-based PhU Results

- 882,132 investigated points
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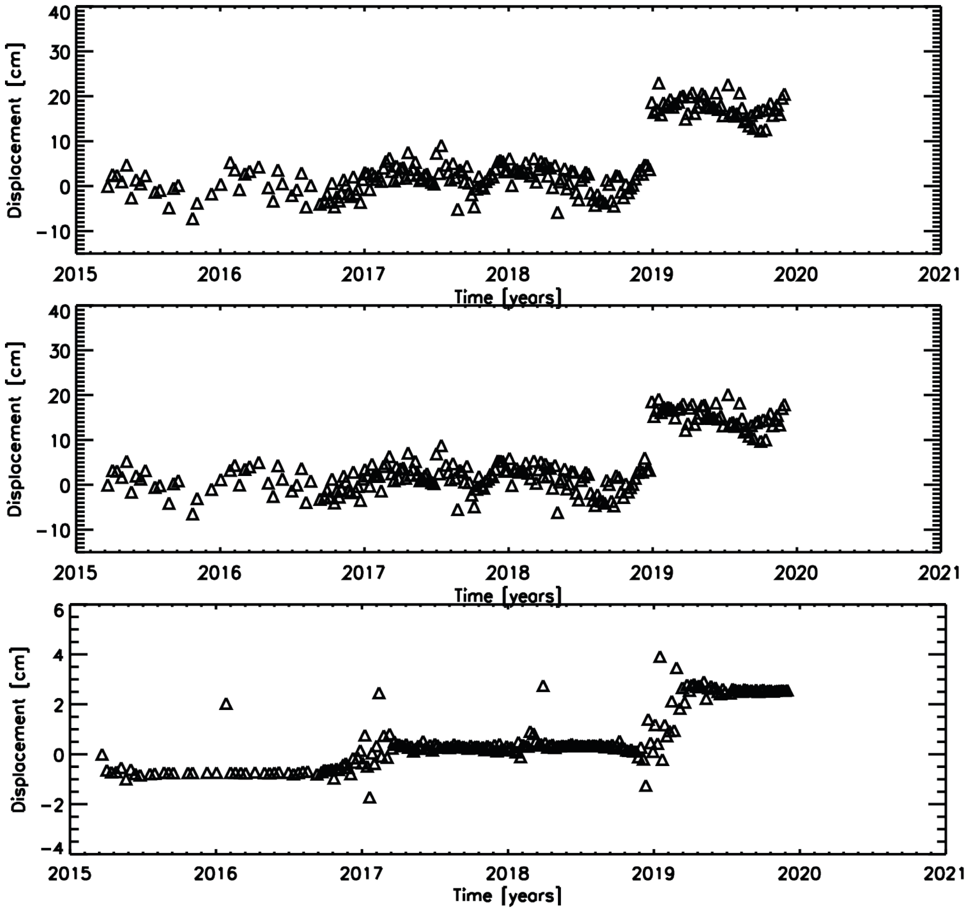

CS-based PhU
Temporal coherence: 0.90

EMCF PhU
Temporal coherence: 0.79

Difference

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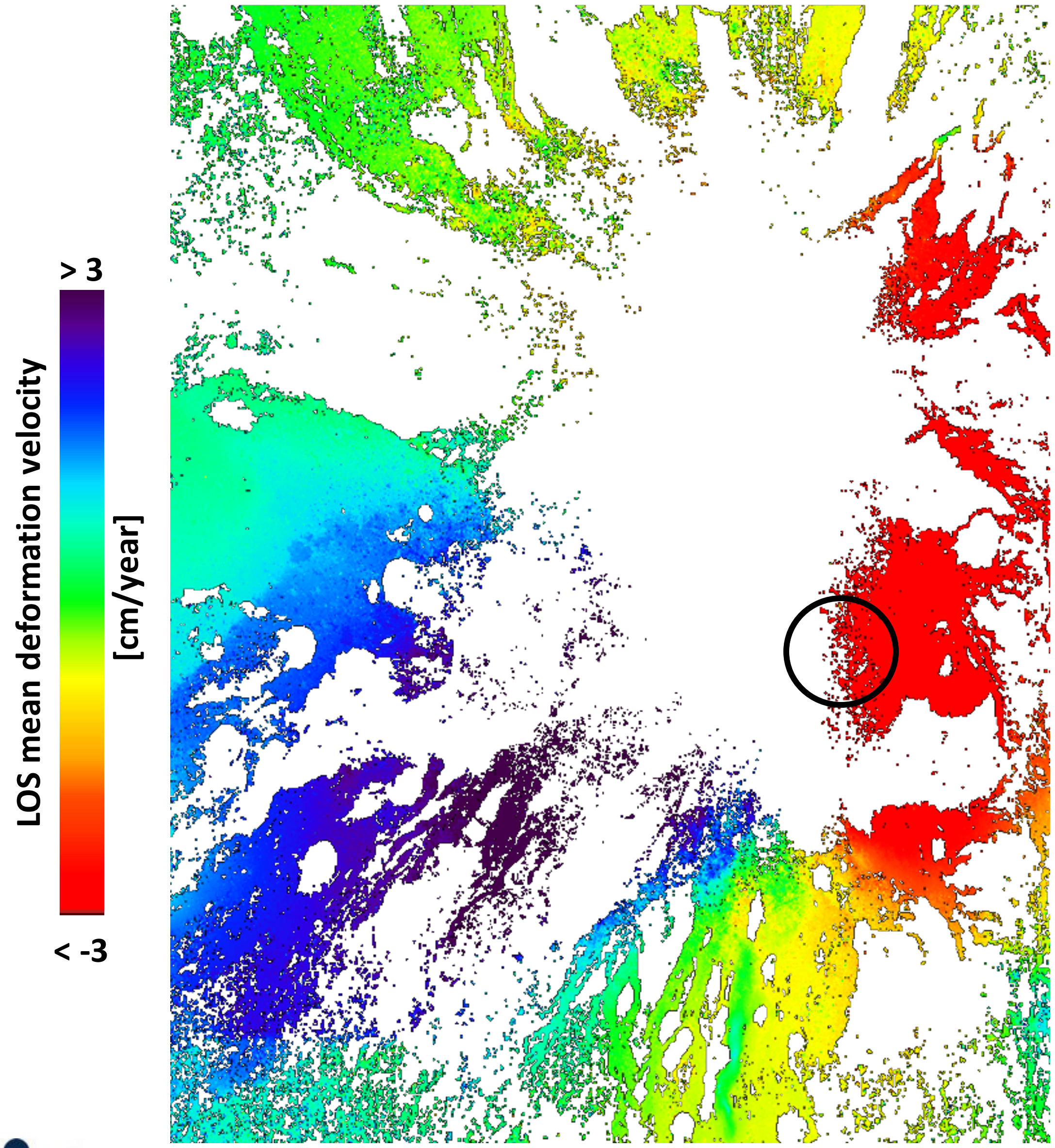
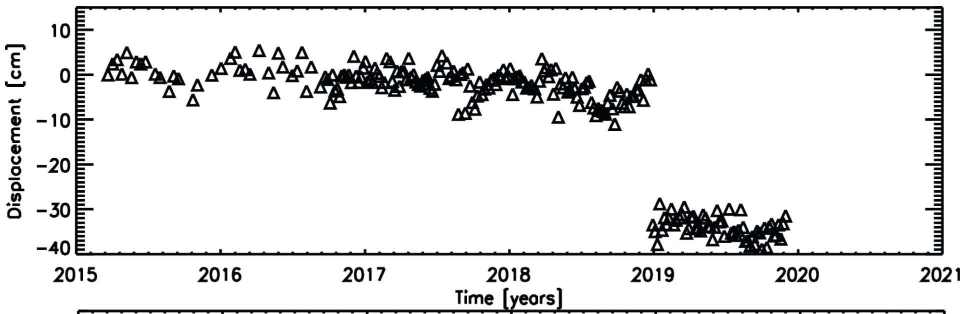
CS-based PhU Results on Sentinel-1 Etna test site

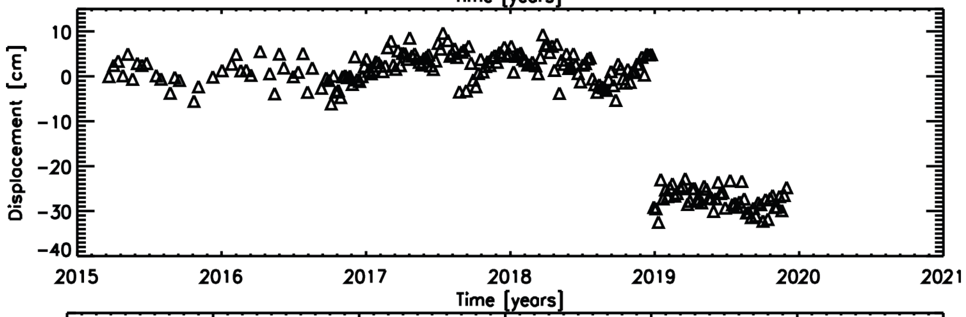


CS-based PhU Results

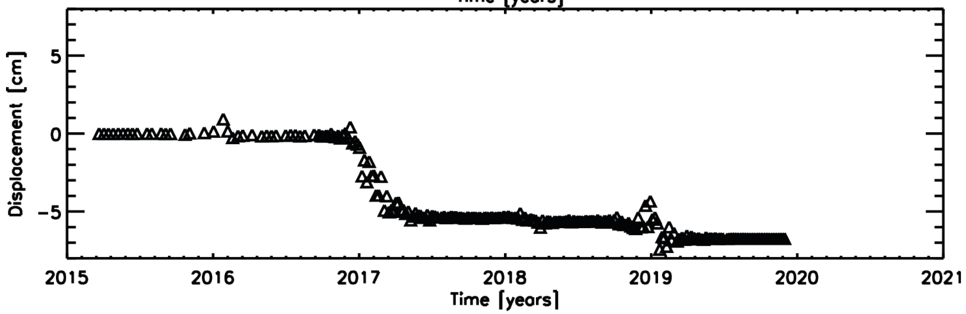
- 882,132 investigated points
- 838,944 coherent points (95.1%) (coherence > 0.9)



CS-based PhU
Temporal coherence: 0.91



EMCF PhU
Temporal coherence: 0.82



Difference

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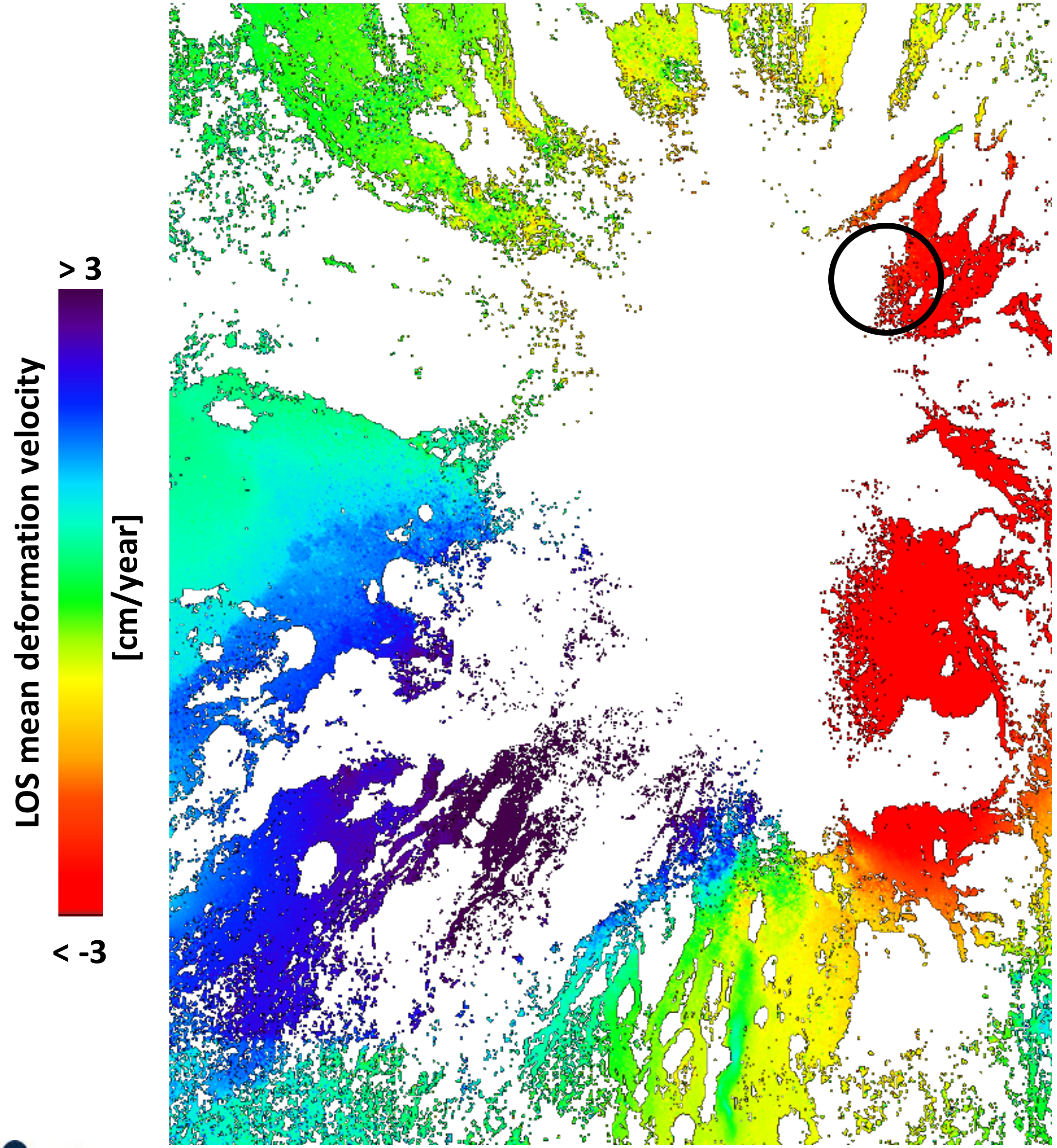
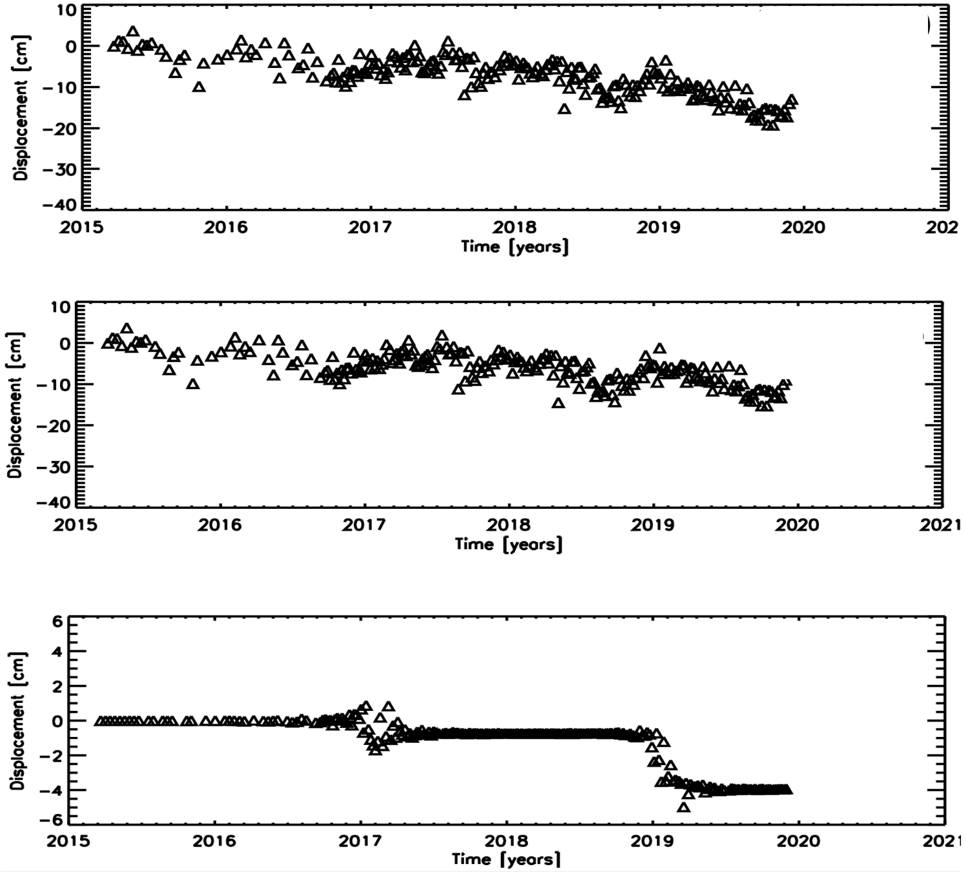
CS-based PhU Results on Sentinel-1 Etna test site



CS-based PhU Results

- 882,132 investigated points
- 838,944 coherent points (95.1%) (coherence > 0.9)



CS-based PhU
Temporal coherence: 0.90

EMCF PhU
Temporal coherence: 0.86

Difference

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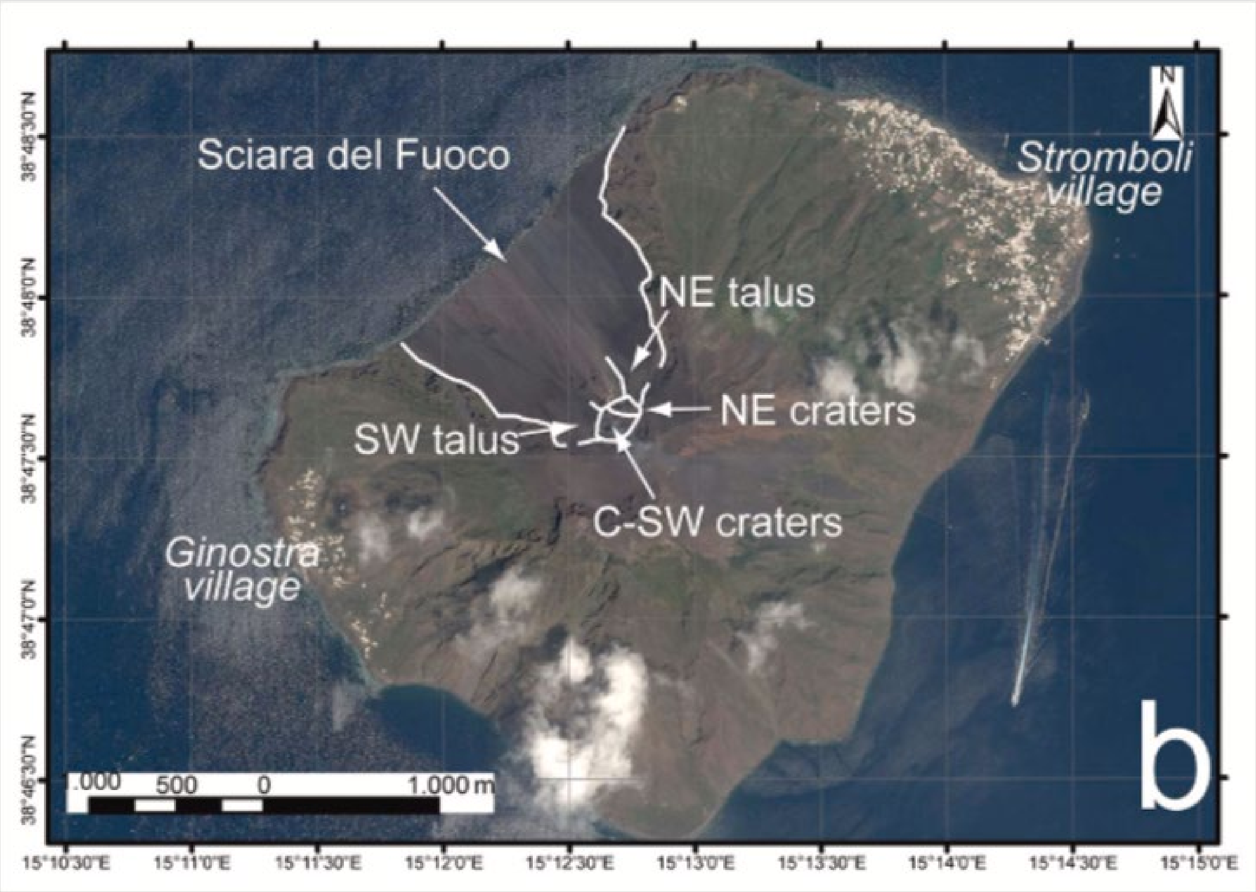
Sentinel-1 Stromboli test site

- ✓ One of the most active volcanoes in Europe.
- ✓ Data acquired by Sentinel-1 (descending orbit, Track 124) between 2016 and 2021.
- ✓ 282 S-1 Acquisitions
- ✓ 801 Interferograms



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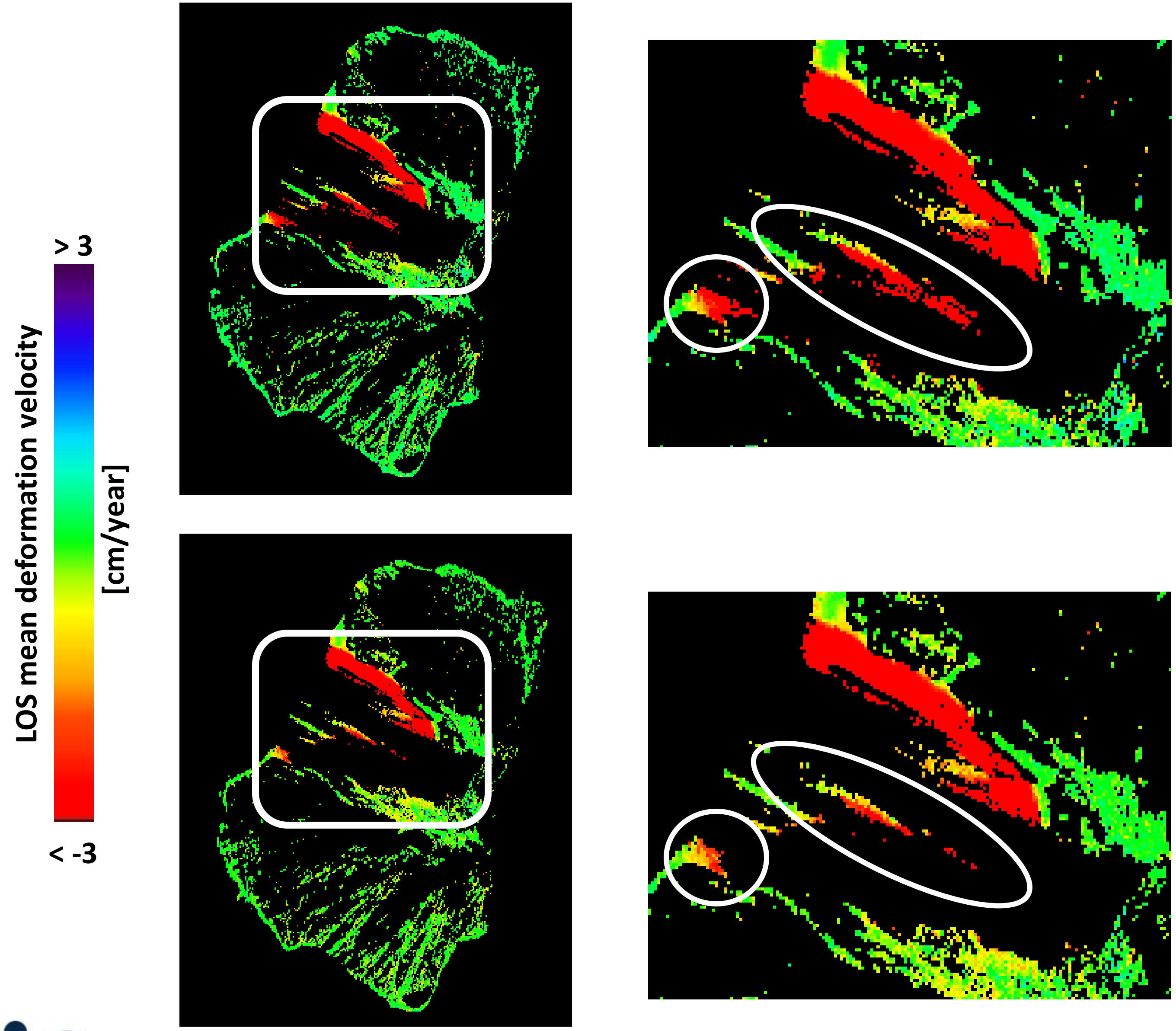
CS-based PhU Results on Sentinel-1 Stromboli test site



CS-based PhU Results

- 12,124 investigated points
- 10,293 coherent points (85%)

EMCF PhU Results

- 12,124 investigated points
- 10,001 coherent points (82%)

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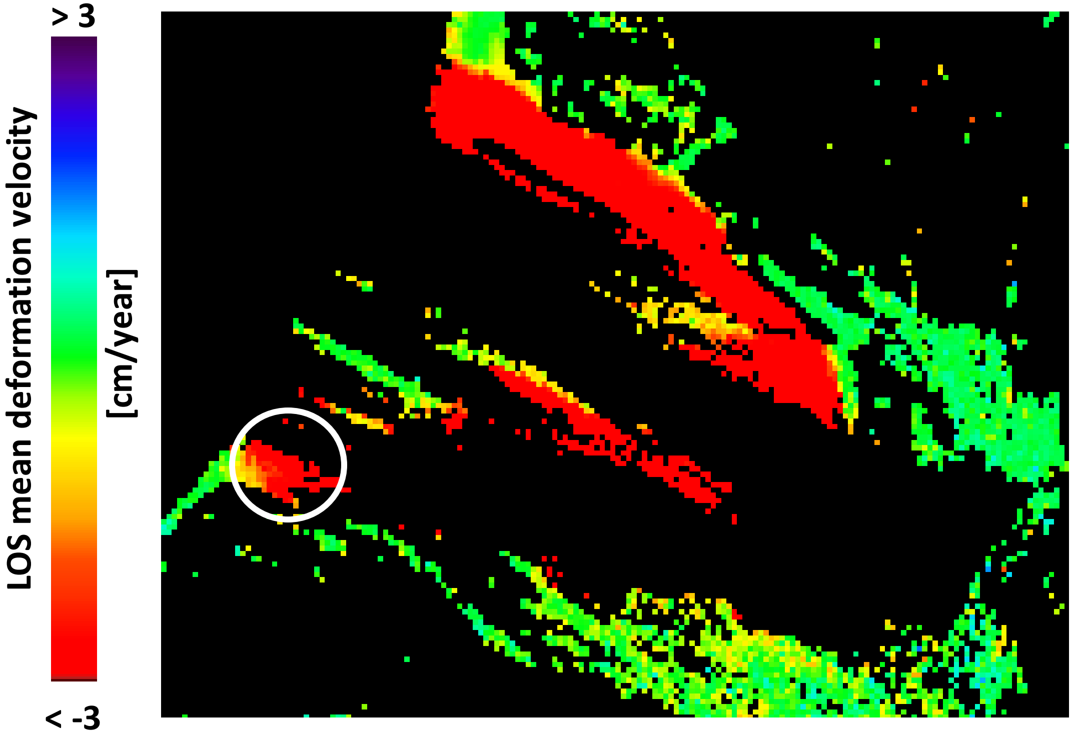
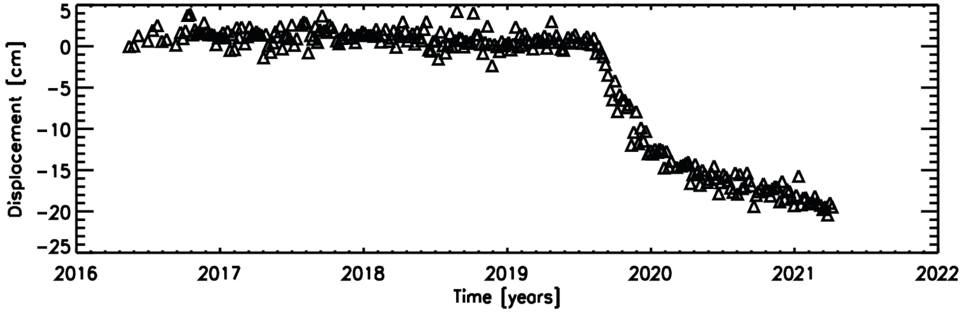
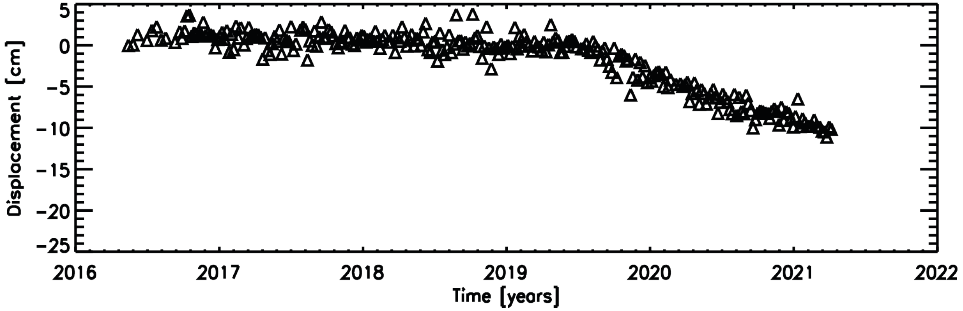
CS-based PhU Results on Sentinel-1 Stromboli test site





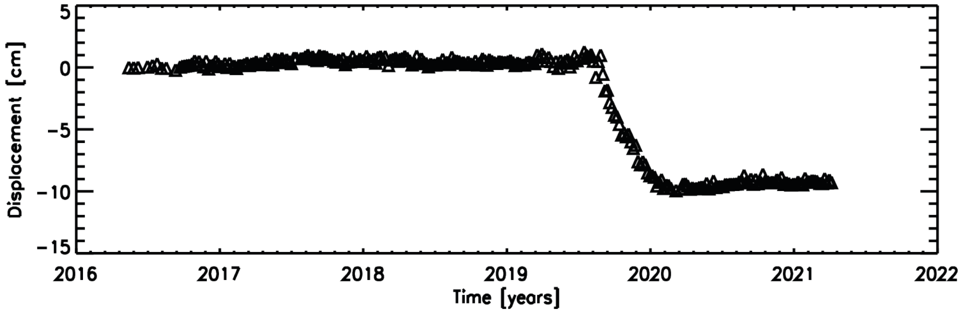
CS-based PhU

Temporal coherence: 0.95



EMCF PhU

Temporal coherence: 0.88



L1-EMCF Difference

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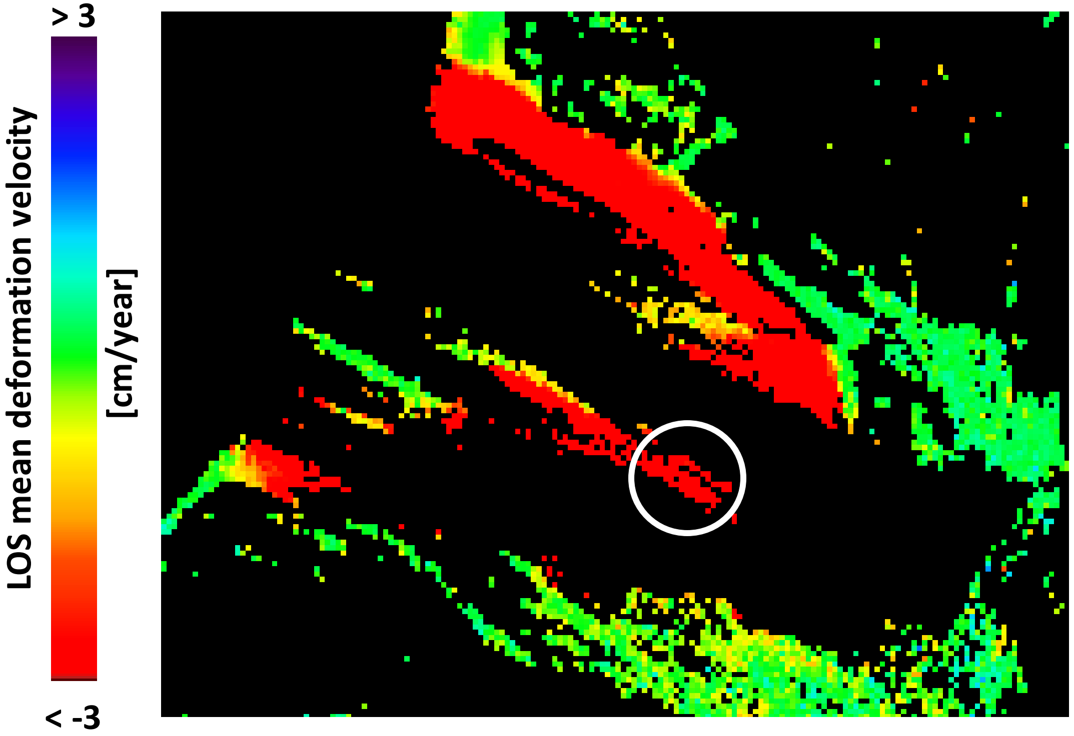
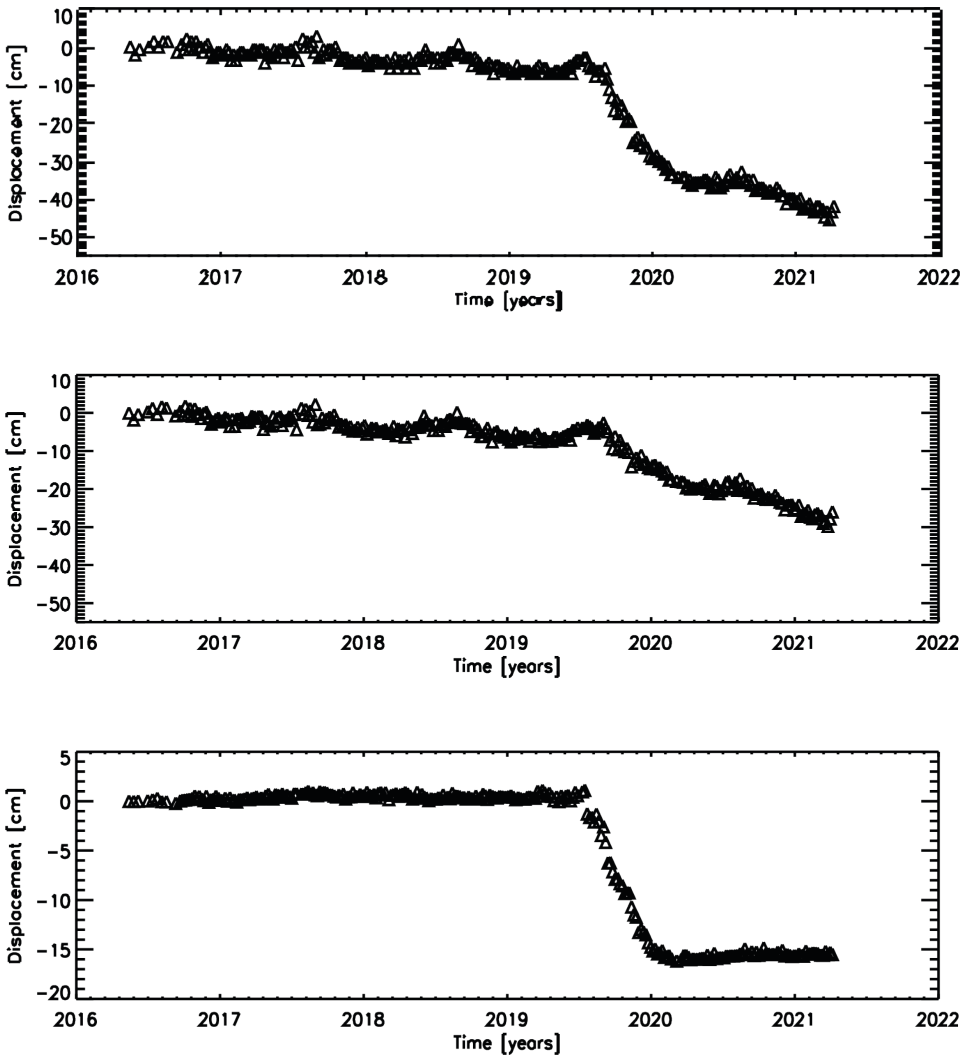
CS-based PhU Results on Sentinel-1 Stromboli test site





CS-based PhU
Temporal coherence: 0.93

EMCF PhU
Temporal coherence: 0.85

L1-EMCF Difference

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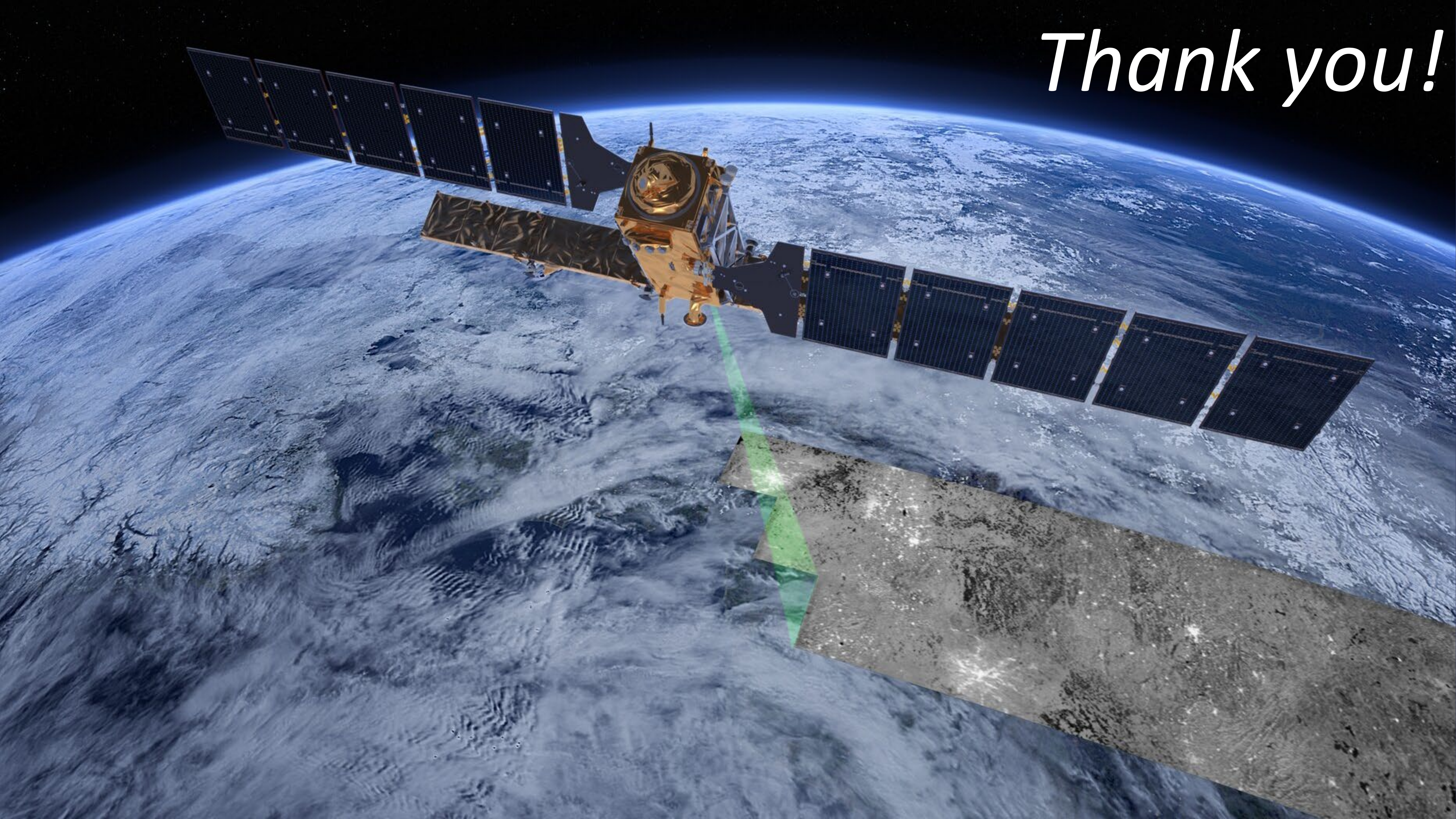
Thank you!