3-D Surface Deformation Performance for Simultaneous Squinted SAR Acquisitions

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Motivation

- Several missions under investigation exploiting two satellites flying simultaneously, one of them operated under a large squint:
  - SESAME, SEntinel-1 SAR Companion Multistatic Explorer (to be submitted to EE9 re-call)
  - Tandem-L (large squint as experimental mode)
  - Parsifal (SAOCOM-CS) (to be submitted to EE9 re-call)

- Current SAR missions have near-polar orbits → blind to N-S motion.

- The second satellite provides a second line of sight (LOS) that can be exploited for the retrieval of the N-S component of the deformation.

- Similar concept as the dual-beam interferometer for ocean surface current vector mapping [1].

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Problem Statement

- **GOAL:** retrieve the 3-D surface deformation asymptotic performance (mean deformation velocity) by combining simultaneous squinted acquisitions.

- Asymptotic performance derived for one LOS by Monti Guarnieri and Tebaldini [2].

- In this work, the mathematical derivation to retrieve the performance is presented considering the correlation of the atmospheric (tropospheric) delays for the simultaneous acquisitions, both for monostatic and bistatic surveys.

- Derivation based on the Hybrid Cramér-Rao Bound (HCRB) (deterministic and random unknowns) similar as in [2].

Problem Statement (cont.)

Correlation length $\sim 1$ km

Height of turbulent troposphere $\sim 1$ km (boundary layer)

$\Delta x \sim 1 \text{ km} \cdot \tan \beta$

Time lapse: $\frac{r_0 \cdot \tan \beta}{v_g}$
Derivation of the Asymptotic Performance for the Monostatic Case

- The differential InSAR phase (ignoring residual DEM error) for a given resolution cell is given by

\[
\begin{align*}
\varphi_A &= \frac{4\pi}{\lambda} \cdot [\langle \vec{x}, \hat{e}_A \rangle + \alpha_{A,t_1} - \alpha_{A,t_2}] + n_A \\
\varphi_B &= \frac{4\pi}{\lambda} \cdot [\langle \vec{x}, \hat{e}_B \rangle + \alpha_{B,t_1} - \alpha_{B,t_2}] + n_B
\end{align*}
\]

- We build an orthonormal basis by generating the sum and difference interferograms:

\[
\begin{align*}
\Delta \varphi_\Sigma &= \varphi_A + \varphi_B = \frac{4\pi}{\lambda} \cdot [\langle \vec{x}, \hat{e}_A \rangle + \langle \vec{x}, \hat{e}_B \rangle + \alpha_{A,t_1} + \alpha_{B,t_1} - (\alpha_{A,t_2} + \alpha_{B,t_2})] + n_A + n_B \\
\Delta \varphi_\Delta &= \varphi_A - \varphi_B = \frac{4\pi}{\lambda} \cdot [\langle \vec{x}, \hat{e}_A \rangle - \langle \vec{x}, \hat{e}_B \rangle + \alpha_{A,t_1} - \alpha_{B,t_1} - (\alpha_{A,t_2} - \alpha_{B,t_2})] + n_A - n_B
\end{align*}
\]

- **Note:** The sum and difference interferograms are uncorrelated both in terms of scatterer phase noise and atmospheric signal, i.e., \( E{\Delta \varphi_\Sigma \cdot \Delta \varphi_\Delta} = 0 \)
Derivation of the Asymptotic Performance for the Monostatic Case (cont.)

• We evaluate the power of the atmosphere for the correlated components in the sum and the difference

$$\sigma_{\alpha,\Sigma}^2 = E \left\{ |\alpha_{A,t_1} + \alpha_{B,t_1}|^2 \right\} = 2 \cdot [R(0) + R(\Delta x)]$$

$$\sigma_{\alpha,\Delta}^2 = E \left\{ |\alpha_{A,t_1} - \alpha_{B,t_1}|^2 \right\} = 2 \cdot [R(0) - R(\Delta x)]$$

• Observations:
  - Difference interferogram mainly oriented to the along-track direction.
  - Depending on the autocorrelation function of the atmosphere, the difference interferogram could be free of atmospheric signal, i.e.,
    $$2 \cdot [R(0) - R(\Delta x)] \to 0 \implies \text{The measurement of the N-S component will benefit from this high correlation between the two lines of sight!}$$
Troposphere Modelling

- Due to the low effective height of the boundary layer (<1 km), the turbulent tropospheric delays are highly correlated, even for large squint differences.

- E.g., for a squint of 20 degrees and with a height of 1 km for the boundary layer, the autocorrelation function would be evaluated at 360 m.

Troposphere Modelling

• We model the boundary layer with Kolmogorov’s power spectrum in the form $1/f^\beta$ (with $\beta = -8/3$ for a 1D profile in the 2D surface) [4]

• Assumed parameters:
  ▪ 1 km correlation length
  ▪ Tropospheric signal power of 1 cm$^2$

• As can be noted, even for large squint angles the signal is highly correlated (squint $10^\circ - 20^\circ$ correspond to $\Delta x = 176 \text{ m} - 360 \text{ m}$)

Comment on the Ionosphere

• The ionosphere cannot be assumed correlated between lines of sight due to its higher altitude.

• In practical cases, the (differential) phase gradient introduced by the ionosphere will be sensed.

• Data-based estimation and correction methods (e.g., split-bandwidth) can be used. For smooth gradients large averaging windows will improve the estimation performance.

• At local scales, the impact of the ionosphere should be minor (assuming smooth gradients).
Atmosphere Evaluation with TSX Staring Spotlight Data

- Generation of sub-looks (2.5 kHz) at increasing Doppler (angular) distances:

  Full staring spotlight bandwidth \( \sim 40 \text{ kHz (4.4°)} \)

- 7 differential interferograms (spectral diversity phases = MAI phases = Multi-Squint phases) with angular distances

\[ [0.29°, 0.87°, 1.44°, 2.02°, 2.66°, 3.18°, 3.76°] \]
Atmosphere Evaluation with TSX Staring Spotlight Data

Chuquicamata mine
Ascending
Master: 2014-06-01
Slave: 2014-06-12
Atmosphere Evaluation with TSX Staring Spotlight Data

Chile
Ascending
Master: 2014-02-11
Slave: 2014-02-22
Atmosphere Evaluation with TSX Staring Spotlight Data

Spain (close to Zaragoza)
Descending
Master: 2014-10-28
Slave: 2014-11-19
Atmosphere Evaluation with TSX Staring Spotlight Data

Atacama Desert Descending
Master: 2014-07-20
Slave: 2014-08-22

\[ \Delta \beta = 0.29^\circ \]
Atacama Desert Descending Pursuit monostatic 10 sec. Acq. date: 2015-02-25

\[ \Delta \beta = 0.29^\circ \]
Derivation of the Asymptotic Performance for the Monostatic Case (cont.)

- Let’s consider now a stack of interferograms, so we have a stack of sum interferograms and a stack of difference interferograms.
- The Fisher Information Matrix (FIM) related to the scatterer phase information for each of these stacks is [2] (distributed scatterers implicitly assumed):

  \[
  \begin{align*}
  \mathbf{X}_{\{\Sigma, \Delta\}} &= \mathcal{N} (\mathbf{\Gamma} \circ \mathbf{\Gamma}^{-1} - \mathbf{I}) \\
  \end{align*}
  \]

  where \( \mathbf{t} \) contains the acquisition times, e.g., in days.
- The first element of the inverse of \( \mathbf{J} \) contains the CRB of the deformation velocity in units radians per day, indicated as \( \sigma_{\Sigma}^2 \) and \( \sigma_{\Delta}^2 \) for the sum and the difference, respectively.

Derivation of the Asymptotic Performance for the Monostatic Case (cont.)

- The inversion from LOS to 3-D motion is given by [5]
  \[ y = Kx \]

  \[
  \begin{bmatrix}
  v_{\text{LOS},1} \\
  \vdots \\
  v_{\text{LOS},N}
  \end{bmatrix} =
  \begin{bmatrix}
  \hat{e}_{e,1} & \hat{e}_{n,1} & \hat{e}_{v,1} \\
  \vdots & \vdots & \vdots \\
  \hat{e}_{e,M} & \hat{e}_{n,M} & \hat{e}_{v,M}
  \end{bmatrix}
  \begin{bmatrix}
  v_e \\
  v_n \\
  v_v
  \end{bmatrix}
  \]

  with the performance of the retrievals given by the diagonal elements of the covariance matrix

  \[
  \sigma_{\{e,n,v\}}^2 = \text{diag}\{(K^T WK)^{-1}\}
  \]

Derivation of the Asymptotic Performance for the Monostatic Case (cont.)

• Assuming one ascending and one descending configuration (4 measurements) we have the following for \( \mathbf{K} \) and \( \mathbf{W} \):

\[
\mathbf{K} = \frac{4\pi}{\lambda} \begin{bmatrix}
\hat{e}_{A,Asc} + \hat{e}_{B,Asc} \\
\hat{e}_{A,Asc} - \hat{e}_{B,Asc} \\
\hat{e}_{A,Desc} + \hat{e}_{B,Desc} \\
\hat{e}_{A,Desc} - \hat{e}_{B,Desc}
\end{bmatrix}
\]

\[
\mathbf{W} = \text{diag} \left\{ \frac{1}{\sigma_{\Sigma,Asc}^2}, \frac{1}{\sigma_{\Delta,Asc}^2}, \frac{1}{\sigma_{\Sigma,Desc}^2}, \frac{1}{\sigma_{\Delta,Desc}^2} \right\}
\]

• \( \mathbf{W} \) is diagonal since we have orthogonalized the system (no correlation between the sum and difference interferograms).
Derivation of the Asymptotic Performance for the Bistatic Case

• In this case, the “trick” of orthogonalizing the system with the sum and the difference interferograms does not work, since atmosphere signal is correlated

• Joint derivation of the performance simultaneously:
  ▪ Two-step approach: In 2D (slant-range and azimuth) for every orbit configuration (asc/desc) and then go to 3D similar as in the monostatic case.
  ▪ Single step: In 3D (local coordinates) with all geometries simultaneously.

• The derivation is an interesting mathematical exercise…
Derivation of the Asymptotic Performance for the Bistatic Case (cont.)

Prats-Iraola, Lopez-Dekker, De Zan, Yague-Martinez, Zonno, Rodriguez-Cassola

“Performance of 3-D Surface Deformation Estimation for Simultaneous Squinted SAR Acquisitions”
IEEE TGRS, Submitted and ‘¿to be published’

Effective squint is halved
(sensitivity loss)
Performance Evaluation

- Use of the derived formulation to compute the asymptotic performance.
- Validation of the mathematical derivation using Monte-Carlo simulations (1000 realizations).
- Processing strategy for the evaluation:
  - 4 LOS: ascending A (non-squinted) and B (squinted), descending A and B
  - Each LOS independently:
    - Phase filtering of the phases with the phase linking algorithm [2]
    - Estimation of the mean deformation velocity
  - 3-D inversion with combined ascending and descending stacks (4 stacks in total)

<table>
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<th>Parameter</th>
<th>System #1</th>
<th>System #2</th>
<th>System #3</th>
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<td>Wavelength</td>
<td>L-band</td>
<td>L-band</td>
<td>C-band</td>
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<tr>
<td>Slave operation</td>
<td>Monostatic</td>
<td>Bistatic</td>
<td>Bistatic</td>
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<td>BL height</td>
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<td>$\sigma_{iono}$</td>
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<td>Repeat-pass cycle</td>
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<td>Incidence angle</td>
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<tr>
<td># Passes</td>
<td></td>
<td>1 Asc.&amp;1 Desc.</td>
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</tr>
</tbody>
</table>

Performance Evaluation

L-band monostatic

L-band bistatic

The diamonds are the result of the Monte-Carlo simulations.
Performance Evaluation

C-band bistatic with ionosphere

C-band bistatic without ionosphere

3D Accuracy [mm/year]

Squint angle [deg]

10 km

320 km

10 km

320 km
Observations

- The retrieval of the N-S component is practically insensitive to the atmospheric phase screen (APS).
- The performance in the retrieval of the N-S component gets closer to the other two for higher APS powers.
- Better performance for the monostatic case ($4\pi$ instead of $2\pi$).
- Better performance in N-S for higher frequency bands, as the troposphere signal is almost cancelled in the difference interferogram.
- Ionosphere remains a limiting factor, since for increasing squint angles the ionospheric signal cannot be considered correlated.
Summary

- The mathematical framework to derive the performance of the 3-D surface deformation (mean velocity) for simultaneous squinted acquisitions has been presented and validated with Monte-Carlo simulations.

- The derivation exploits the HCRB, which considers a certain correlation in the atmospheric signal between the simultaneous acquisitions.

- Squinted acquisitions are baseline (or thought as experiment) of interferometric SAR missions currently under investigation. The presented formulation is being used to evaluate the mission performance.

- Due to the (quasi-)simultaneity of the acquisitions, the tropospheric signals for both acquisitions are highly correlated, hence improving the performance in the retrieval of the deformation in the N-S direction, while the East-West and vertical components are still affected by the APS.

- The ionosphere might limit the performance, as it can no longer be assumed correlated. Start-of-the-art approaches (e.g., split-bandwidth) could be exploited to reduce the ionospheric signal power prior to the time series processing. Lesser impact at local scales.
Thank you for your attention!